Lecture 7 (week 7: 31 March - 1 April 2025)

Electro-mechanics: elasticity and piezoelectricity

- Symmetry of electro-mechanical response: insight from thermodynamics

- Electromechanical effects

- Stiffness and electromechanics: practical aspects and control of elastic properties

Some planning and important dates

- Tuesday, 15 April – test in class

- **17.06 9:15 Exam, (to be confirmed by SAC)**

Exercise 6.4

At room temperature $BaTiO_3$ is a piezoelectric. For this reason, the application of an electric field to a sample of $BaTiO_3$ leads to its deformation and to a change of its volume. Will the crystal increase or decrease its volume, if the field is directed:

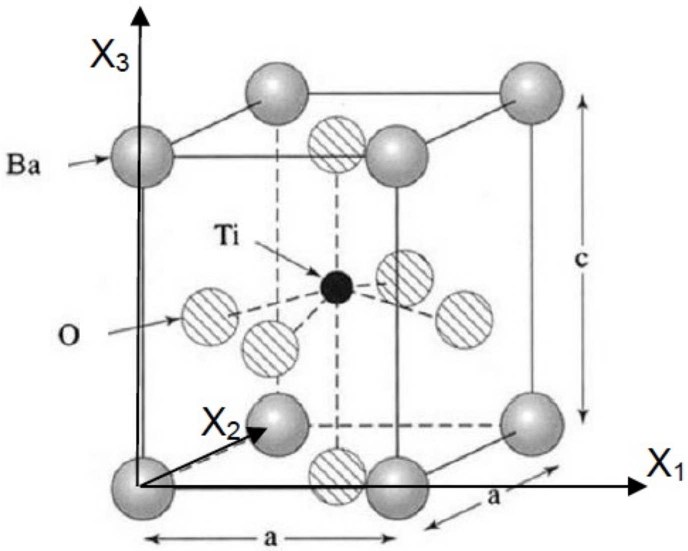
(a) along the $[111]$ direction?

(b) along the $[11\bar{1}]$ direction?

The sample is mechanically free and its temperature is always kept constant. x_3 axis is directed along 4-fold axis (see **Fig.1**). For this reference frame, the piezoelectric tensor of $BaTiO_3$ (point symmetry 4*mm*) has 3 independent components at room temperature: d_{33}= 86 pC/N; d_{31}= - 35 pC/N; d_{15}= 392 pC/N

Additional information: the relative change of volume of a sample deformed with the strain ε_{ij} is equal to its trace ε_{ii}

$\varepsilon_n = d_{in}E_i$ – converse piezoelectric effect.



Exercise 6.4

$\varepsilon_n = d_{in} E_i$ – converse piezoelectric effect.

The piezoelectric tensor for $BaTiO_3$, a perovskite structure with $4mm$ symmetry, for the reference frame described in the problem, is (see Symmetry Tables):

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

The relative variation of volume can be found as: $\frac{\Delta V}{V} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$.

The converse piezoelectric effect gives:

$\varepsilon_1 = d_{i1} E_i = d_{11} E_1 + d_{21} E_2 + d_{31} E_3 = d_{31} E_3$ ($d_{11} = d_{21} = 0$ in the considered symmetry)

$$\varepsilon_2 = d_{i2} E_i = d_{32} E_3 = d_{31} E_3, \qquad \varepsilon_3 = d_{i3} E_i = d_{33} E_3.$$

$$\frac{\Delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = d_{31} E_3 + d_{31} E_3 + d_{33} E_3 = (2 d_{31} + d_{33}) E_3.$$

Exercise 6.4

(a) If the field is directed along the [111], $\vec{E} = \frac{E}{\sqrt{3}}(1,1,1)$, hence

$$\frac{\Delta V}{V} = (2d_{31} + d_{33})\frac{E}{\sqrt{3}} = \frac{E}{\sqrt{3}} \cdot 16\frac{\text{pC}}{\text{N}} > 0$$

Thus, the volume increases.

(b) If the field is directed along the $[11\bar{1}]$, $\vec{E} = \frac{E}{\sqrt{3}}(1,1,-1)$, and

$$\frac{\Delta V}{V} = -(2d_{31} + d_{33})\frac{E}{\sqrt{3}} = -\frac{E}{\sqrt{3}} \cdot 16\frac{\text{pC}}{\text{N}} < 0.$$

In this case, the volume decreases.

Exercise 6.5 – a paper…

PIEZOELECTRICS

Induced giant piezoelectricity in centrosymmetric oxides

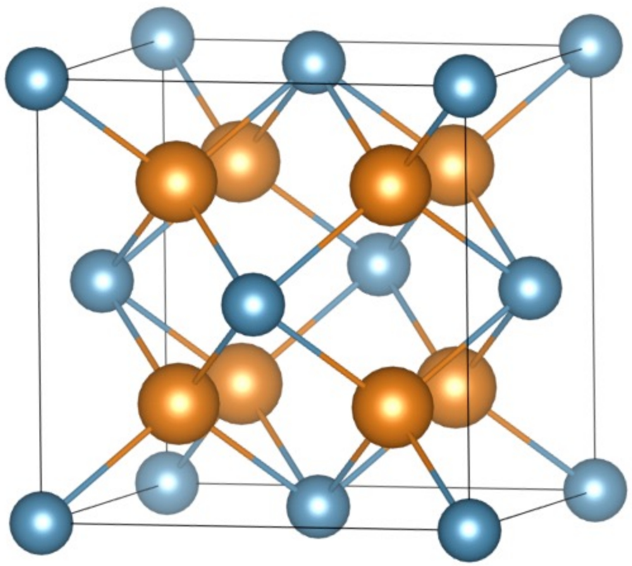
D.-S. Park[1,2*], M. Hadad[2], L. M. Riemer[1], R. Ignatans[3], D. Spirito[4], V. Esposito[5], V. Tileli[3], N. Gauquelin[6,7], D. Chezganov[6,7], D. Jannis[6,7], J. Verbeeck[6,7], S. Gorfman[4], N. Pryds[5], P. Muralt[2], D. Damjanovic[1*]

Piezoelectrics are materials that linearly deform in response to an applied electric field. As a fundamental prerequisite, piezoelectric materials must have a noncentrosymmetric crystal structure. For more than a century, this has remained a major obstacle for finding piezoelectric materials. We circumvented this limitation by breaking the crystallographic symmetry and inducing large and sustainable piezoelectric effects in centrosymmetric materials by the electric field–induced rearrangement of oxygen vacancies. Our results show the generation of extraordinarily large piezoelectric responses [with piezoelectric strain coefficients (d_{33}) of ~200,000 picometers per volt at millihertz frequencies] in cubic fluorite gadolinium-doped CeO_{2-x} films, which are two orders of magnitude larger than the responses observed in the presently best-known lead-based piezoelectric relaxor–ferroelectric oxide at kilohertz frequencies. These findings provide opportunities to design piezoelectric materials from environmentally friendly centrosymmetric ones.

Park *et al.*, *Science* **375**, 653–657 (2022)   11 February 2022

Exercise 6.5 – a paper…

Fluorite: CeO_2 – layers of Oxygen – favorable conditions for ionic conduction



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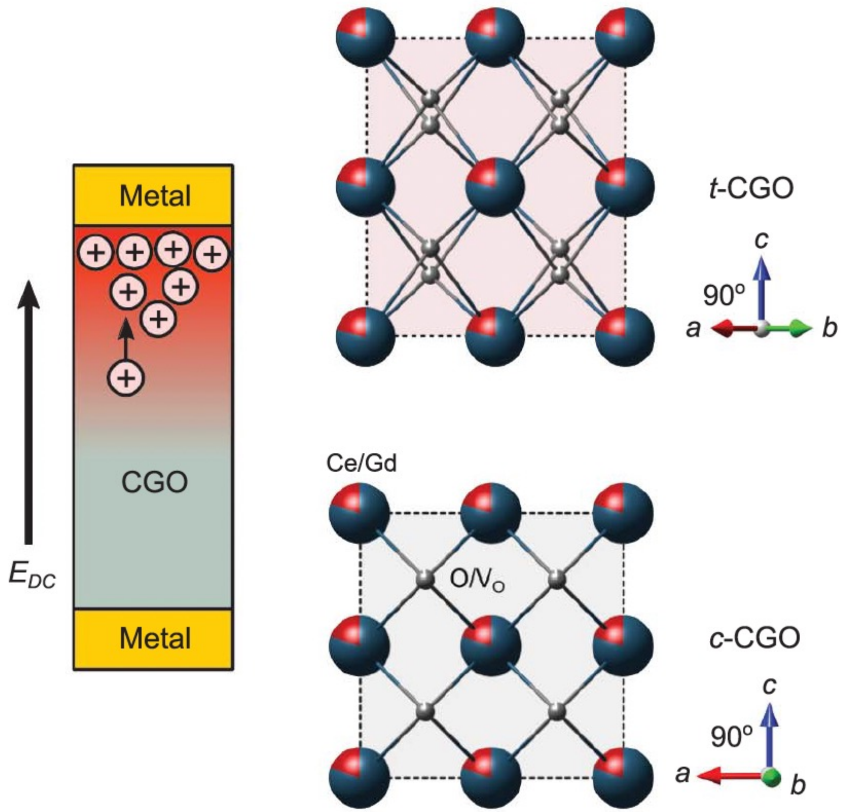


Formation of oxygen vacancies – chemical expansion – change of crystalline structure

Park *et al.*, *Science* **375**, 653–657 (2022)   11 February 2022

Exercise 6.5 – a paper…

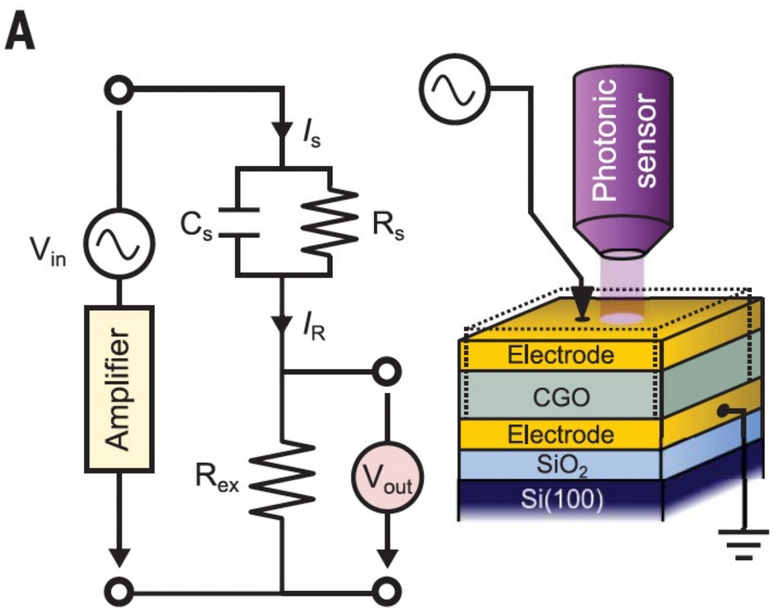
cidated, but the electric field–induced mechanical deformation is certainly related to the presence and short-range motion of oxygen vacancies (*12–14*).

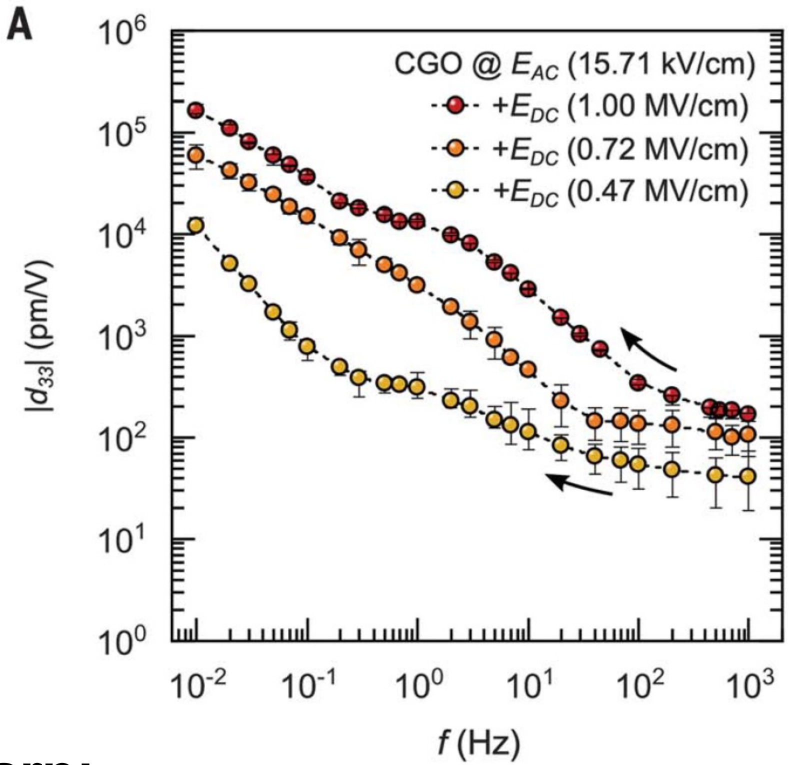
We demonstrate a paradigm shift for achieving large, electric field–induced piezoelectricity in centrosymmetric materials. We show that for Gd-doped CeO_{2-x} (CGO) films, which have a cubic fluorite centrosymmetric structure, we can achieve very high values of the electric field–induced piezoelectric strain (x ~ 26%) and apparent longitudinal piezoelectric coefficients (d_{33} of ~200,000 pm/V). This latter value, measured in the millihertz range, is two to three orders of magnitude larger than that observed in the best piezoelectric perovskite oxides—e.g., $Pb(Mg_{1/3}Nb_{2/3})O_3$-$PbTiO_3$ with $d_{33} \approx 2000$ pm/V (*3*). Notably, and relevant for applications, the induced effect is comparable to that in the best PZT thin films (~100 pm/V) in the kilohertz range (*15*). We argue that the change in the strain mechanism from the short-range lattice or ionic defect–based mechanism above 10 Hz to the one at low frequencies, is based on distinct actions of the long-range migration of ions (oxygen vacancies, V_O) and electrons. Our results show that the electric field–induced redistribution of mobile charges in the films leads to crystal phase transition, associated with chemical expansion, and material heterogeneity. These combined effects result in giant piezoelectric and electrostrictive responses and point toward a previously unknown electromechanical mechanism in centrosymmetric fluorites and materials with large ionic and electronic conductivity in general.

We deposited polycrystalline $(Gd_{0.2}Ce_{0.8})O_{2-x}$ films on $Al/SiO_2/Si(100)$ substrate at room temperature by sputter deposition (Fig. 1A). The CGO films had thicknesses in the range of ~1.25 to ~1.8 μm (fig. S1) (*16*). The electrostrictive strain for a sample of length L is defined as

$$x = \Delta L/L = ME^2 \quad (1)$$

Exercise 6.5 – a paper…





Summary and takeaway, what we learn:

- **Centrosymmetric materials are not piezoelectric, except for…**
- **If you break the central symmetry by stress or defect engineering the material may become piezoelectric**
- **Ionics is a powerful tool for tailoring the properties of oxides**
- **A high electric field may alter the properties**
- **Pay attention to the relevant frequency range!**

Todays topics:

- Symmetry of electro-mechanical response: insight from thermodynamics

- Electromechanical effects & boundary conditions

- Stiffness and electromechanics: practical aspects and control of elastic properties

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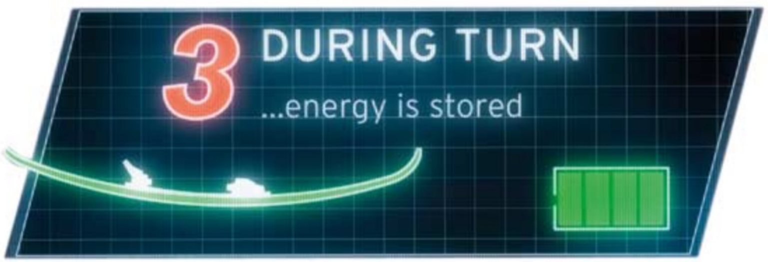

PIEZOELECTRIC FIBERS TRANSFORM KINETIC ENERGY INTO ELECTRICAL ENERGY WHICH IS STORED. ELECTRICAL ENERGY IS IMMEDIATELY RELEASED TO AREAS OF THE SKI, WHERE ADDITIONAL ENERGY IS REQUESTED. TIMING AND RELEASE ARE AUTOMATICALLY CONTROLLED AND COORDINATED. DEPENDING ON THE FLEX PATTERN OF DIFFERENT SKI MODELS, SENSORS ARE PROGRAMMED BEFOREHAND: THE MORE AGGRESSIVE THE SKI HAS TO BE, THE STIFFER THE TAIL WILL BECOME.

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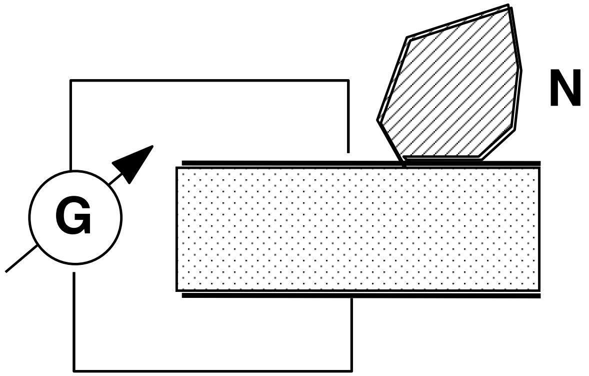
Does this make sense? What is the materials physics behind this effect?

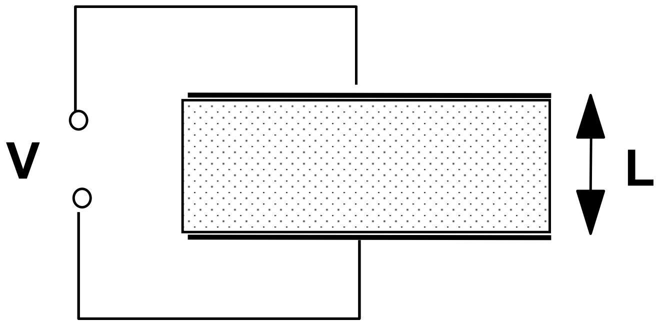
Converse and direct piezoelectric effect



$$\frac{Q}{N} = d$$ (pC/N)

direct piezoelectric effect

$$d^* = d$$



$$\frac{\Delta L}{V} = d^*$$ (pm/V)

converse piezoelectric effect

$$D_i = d_{ijk}\sigma_{jk}$$

$$\varepsilon_{jk} = d^*_{ijk} E_i$$

$$d^*_{ijk} = d_{ijk}$$

experimental findings:
remarkable symmetry of the coefficients:

$$d^*_{ijk} = d_{ijk}$$ **Electromechanical**

$$K_{ij} = K_{ji}$$ **Dielectric**

$$i = 1-3$$

$$s_{nm} = s_{mn} \qquad c_{nm} = c_{mn}$$ **Elastic**

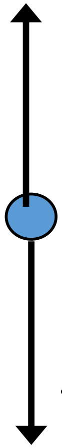
$$n = 1-6$$

Why symmetry??

All these remarkable experimental findings
can be explained
taking into account
the energy aspect of the problem

Energetics of mechanics

Material point



$\vec{r}$ **displacement**

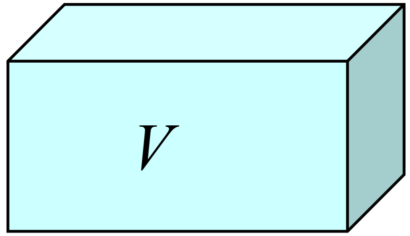
$\vec{f}$ **force**

work

$$A = -\int f_i dr_i$$

$$i = 1 - 3$$

Solid material



V – volume of sample

$$A = V \int \sigma_n d\varepsilon_n \qquad n = 1 - 6$$

Energy conservation law

energy change $\Delta U = A$

energy density change $\Delta W = \dfrac{\Delta U}{V} = \dfrac{A}{V}$

$$dU = -f_i dr_i \qquad f_i = -\frac{\partial U}{\partial r_i}$$

$$dW = \sigma_n d\varepsilon_n \qquad \sigma_n = \frac{\partial W}{\partial \varepsilon_n}$$

Symmetry of c_{nm}

$$\sigma_n = \frac{\partial W}{\partial \varepsilon_n}$$

$$\sigma_n = c_{nm}\varepsilon_m \quad \Rightarrow \quad c_{nm} = \frac{\partial \sigma_n}{\partial \varepsilon_m} = \frac{\partial}{\partial \varepsilon_m}\frac{\partial W}{\partial \varepsilon_n} = \frac{\partial^2 W}{\partial \varepsilon_m \partial \varepsilon_n}$$

$$\frac{\partial^2 W}{\partial \varepsilon_m \partial \varepsilon_n} = \frac{\partial^2 W}{\partial \varepsilon_n \partial \varepsilon_m} \quad \Rightarrow \quad c_{nm} = c_{mn}$$

!!!!

Maxwell relations

Symmetry of s_{nm} and Legendre transformation

$$dW = \sigma_n d\varepsilon_n \qquad \tilde{W} = W - \sigma_n \varepsilon_n$$

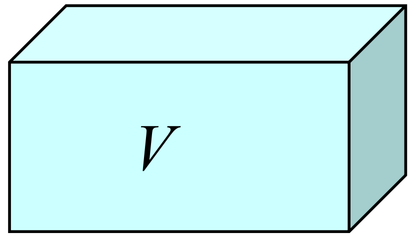
$$d\tilde{W} = \sigma_n d\varepsilon_n - \sigma_n d\varepsilon_n - \varepsilon_n d\sigma_n = -\varepsilon_n d\sigma_n \qquad \varepsilon_n = -\frac{\partial \tilde{W}}{\partial \sigma_n}$$

$$\varepsilon_n = s_{nm}\sigma_m \Rightarrow s_{nm} = \frac{\partial \varepsilon_n}{\partial \sigma_m} = -\frac{\partial}{\partial \sigma_m}\frac{\partial \tilde{W}}{\partial \sigma_n} = -\frac{\partial^2 \tilde{W}}{\partial \sigma_m \partial \sigma_n}$$

$$\frac{\partial^2 \tilde{W}}{\partial \sigma_m \partial \sigma_n} = \frac{\partial^2 \tilde{W}}{\partial \sigma_n \partial \sigma_m} \Rightarrow s_{nm} = s_{mn}$$

Energetics of electrostatics

$$A = V\int E_i dD_i$$ **work**



$$i = 1 - 3$$

V – volume of sample

Energy conservation law

energy density change $$\Delta W = A/V$$

$$dW = E_i dD_i$$

$$E_i = \frac{\partial W}{\partial D_i}$$

Symmetry of K^{-1}_{nm}

$$E_i = \frac{\partial W}{\partial D_i}$$

$$D_i = \varepsilon_0 K_{ij} E_j$$

$$K^{-1}_{ij} = \varepsilon_0 \frac{\partial E_j}{\partial D_i}$$

$$\Rightarrow \quad K^{-1}_{ij} = \varepsilon_0 \frac{\partial}{\partial D_i} \frac{\partial W}{\partial D_j} = \varepsilon_0 \frac{\partial^2 W}{\partial D_i \partial D_j}$$

$$\frac{\partial^2 W}{\partial D_i \partial D_j} = \frac{\partial^2 W}{\partial D_j \partial D_i} \quad \Rightarrow \quad K^{-1}_{ij} = K^{-1}_{ji}$$

Symmetry of K_{nm}

$$dW = E_i dD_i \qquad \tilde{\tilde{W}} = W - E_i D_i$$

$$d\tilde{\tilde{W}} = E_i dD_i - E_i dD_i - D_i dE_i = -D_i dE_i \qquad D_i = -\frac{\partial \tilde{\tilde{W}}}{\partial E_i}$$

$$D_i = \varepsilon_0 K_{ij} E_j$$

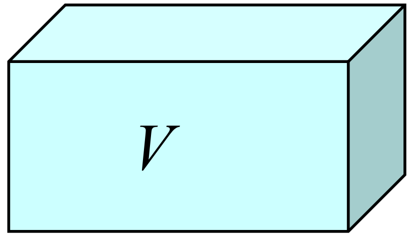
$$K_{ij} = \frac{1}{\varepsilon_0}\frac{\partial D_i}{\partial E_j} \quad \Rightarrow \quad K_{ij} = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial E_j}\frac{\partial \tilde{\tilde{W}}}{\partial E_i} = -\frac{1}{\varepsilon_0}\frac{\partial^2 \tilde{\tilde{W}}}{\partial E_j \partial E_i}$$

$$\frac{\partial^2 \tilde{\tilde{W}}}{\partial E_i \partial E_j} = \frac{\partial^2 \tilde{\tilde{W}}}{\partial E_j \partial E_i} \quad \Rightarrow \quad K_{ij} = K_{ji}$$

Energetics of electro-mechanics

$$A = V\int\left(E_i dD_i + \sigma_n d\varepsilon_n\right)$$

work



$$i = 1-3 \qquad n = 1-6$$

Energy conservation law

$$\Delta W = A/V$$

$$dW = E_i dD_i + \sigma_n d\varepsilon_n \qquad E_i = \frac{\partial W}{\partial D_i} \qquad \sigma_n = \frac{\partial W}{\partial \varepsilon_n}$$

$$W^* = W - E_i D_i - \sigma_n \varepsilon_n$$

$$dW^* = -D_i dE_i - \varepsilon_n d\sigma_n$$

$$D_i = -\frac{\partial W^*}{\partial E_i} \qquad \varepsilon_n = -\frac{\partial W^*}{\partial \sigma_n}$$

Relation of $d^*_{in} = d_{in}$

$$D_i = d_{in}\sigma_n \qquad \varepsilon_n = d^*_{in} E_i$$

$$D_i = -\frac{\partial W^*}{\partial E_i} \qquad \varepsilon_n = -\frac{\partial W^*}{\partial \sigma_n}$$

$$d_{in} = \frac{\partial D_i}{\partial \sigma_n} = -\frac{\partial^2 W^*}{\partial \sigma_n \partial E_i} \qquad d^*_{in} = \frac{\partial \varepsilon_n}{\partial E_i} = -\frac{\partial^2 W^*}{\partial E_i \partial \sigma_n}$$

$$\frac{\partial^2 W^*}{\partial E_i \partial \sigma_n} = \frac{\partial^2 W^*}{\partial \sigma_n \partial E_i} \quad \Rightarrow \quad d^*_{in} = d_{in}$$

Symmetry and constitutive equations of linear electromechanical response

$$D_i = \varepsilon_0 K_{ij} E_j + d_{in} \sigma_n$$

$$\varepsilon_n = d_{in} E_i + s_{nm} \sigma_m$$

The matrix from of the constitutive eq.

$$\begin{pmatrix} D \\ \varepsilon \end{pmatrix} = \underline{\Lambda} \begin{pmatrix} E \\ \sigma \end{pmatrix}$$

This matrix is symmetric

$$\Lambda_{ij} = \begin{pmatrix} \varepsilon_o K_{11} & \varepsilon_o K_{12} & \varepsilon_o K_{13} & d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ \varepsilon_o K_{12} & \varepsilon_o K_{22} & \varepsilon_o K_{23} & d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ \varepsilon_o K_{13} & \varepsilon_o K_{23} & \varepsilon_o K_{33} & d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \\ d_{11} & d_{21} & d_{31} & s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ d_{12} & d_{22} & d_{32} & s_{12} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ d_{13} & d_{23} & d_{33} & s_{13} & s_{23} & s_{33} & s_{34} & s_{35} & s_{36} \\ d_{14} & d_{24} & d_{34} & s_{14} & s_{24} & s_{34} & s_{44} & s_{45} & s_{46} \\ d_{15} & d_{25} & d_{35} & s_{15} & s_{25} & s_{35} & s_{45} & s_{55} & s_{56} \\ d_{16} & d_{26} & d_{36} & s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & s_{66} \end{pmatrix}$$

Thermodynamic potentials and independent variables

Thermodynamic potentials	**Independent variables (a,b)**		
W	$dW = \sigma_n d\varepsilon_n + E_i dD_i$	ε_n	D_i
$\tilde{W} = W - \sigma_n \varepsilon_n$	$d\tilde{W} = -\varepsilon_n d\sigma_n + E_i dD_i$	σ_n	D_i
$\tilde{\tilde{W}} = W - E_i D_i$	$d\tilde{\tilde{W}} = \sigma_n d\varepsilon_n - D_i dE_i$	ε_n	E_i
$W^* = W - E_i D_i - \varepsilon_i \sigma_i$	$dW^* = -\varepsilon_n d\sigma_n - D_i dE_i$	σ_n	E_i

Legendre transformations ↗

Maxwell relations →

$$\frac{\partial^2 W}{\partial a \partial b} = \frac{\partial^2 W}{\partial b \partial a}$$

More information from energy arguments

$$dW = E_i dD_i \qquad D_i = \varepsilon_o K_{ij} E_j$$

$$W = \int E_i dD_i = \frac{\varepsilon_0}{2} E_i K_{ij} E_j = \frac{\varepsilon_0}{2}\left(K^{(1)} E'^2_1 + K^{(2)} E'^2_2 + K^{(3)} E'^2_3\right)$$

Stability

$$K^{(1)} > 0 \quad K^{(2)} > 0 \quad K^{(3)} > 0$$

All coefficients *K* must be positive, otherwise the electric field will spontaneously arise in order to reduce the energy

Limitations of use of thermodynamic potentials: equilibrium conditions

Dielectric response
- equilibrium property

$$D_i = \varepsilon_o K_{ij} E_j$$

$$K_{ij} = K_{ji}$$

Energy at fixed *E*

$$dW = E_i dD_i \qquad W = \frac{1}{2} E_j D_j$$

Electrical Conductivity
- transport property

$$J_i = \tau_{ij} E_j$$

$$\tau_{ij} = \tau_{ji}$$

Energy loss at fixed *E*

$$\frac{dW}{dt} = -E_j J_j$$

Maxwell relations cannot be applied:
the energy does not characterize the state of the material in a transport phenomenon

Onsager relations are used for transport phenomena (to be discussed later)

Linear electro-mechanical response under different conditions

Clamped ← → Mechanically free

Short-circuit ←→ Open-circuit

Dielectric constant

$$D_i = \varepsilon_0 K_{ij} E_j + d_{in} \sigma_n$$

dielectric constant measured in mechanically a free crystal

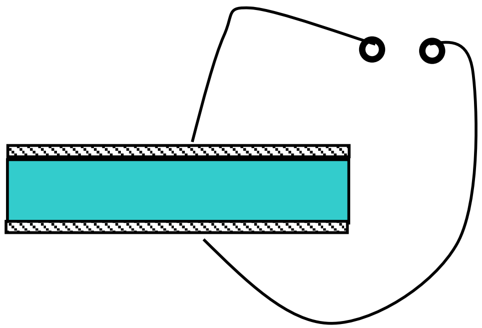
1. Non-piezoelectrics – permittivity is independent of measurements conditions.

2. Piezoelectrics - permittivity is dependent on mechanical measurement conditions.

Dielectric constant of piezoelectrics

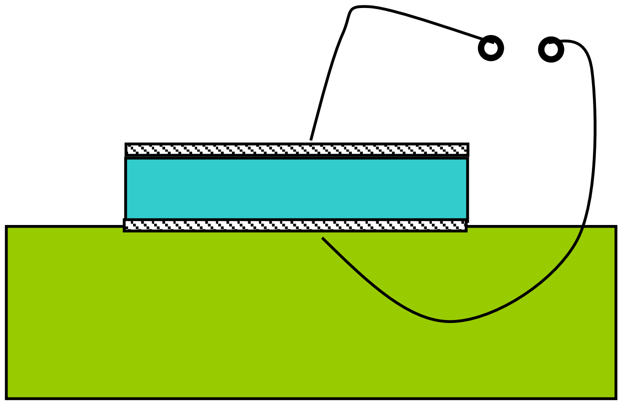
$$D_i = \varepsilon_0 K_{ij} E_j + d_{in}\sigma_n$$

$$\varepsilon_n = d_{in} E_i + s_{nm}\sigma_m$$



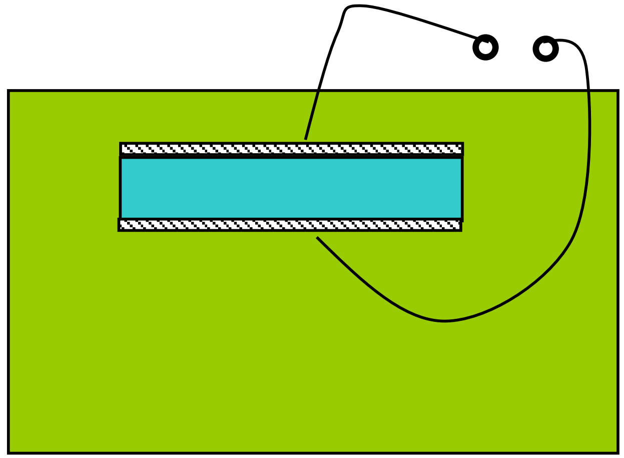
Mechanically free

K_{ij}



Partial clamping with a hard and thick substrate

K_{ij}^{*}



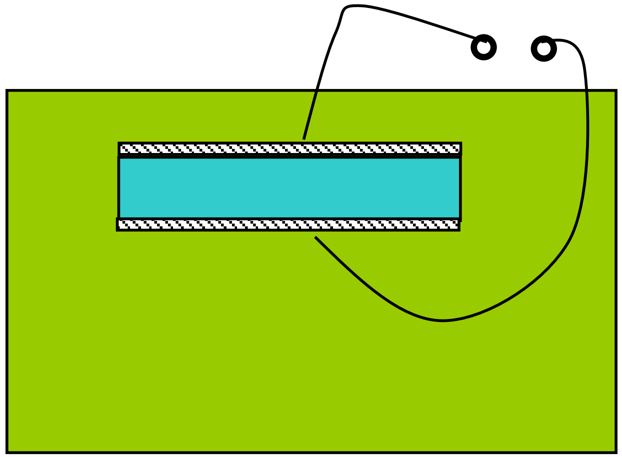
Full clamping with a hard and thick matrix

K_{ij}^{**}

Dielectric constant of piezoelectrics

$$D_i = \varepsilon_0 K_{ij} E_j + d_{in} \sigma_n$$

$$\varepsilon_n = d_{in} E_i + s_{nm} \sigma_m$$



$$\varepsilon_n = 0$$

Full clamping with a hard and thick matrix

$$\sigma_m = -s_{mn}^{-1} d_{in} E_j = -c_{mn} d_{in} E_i$$

$$D_i = \varepsilon_0 K_{ij} E_j - d_{im} c_{mn} d_{jn} E_j$$

$$K_{ij}^{**} = K_{ij} - \varepsilon_0^{-1} d_{im} c_{mn} d_{jn}$$

Effect can be strong (see exercises)

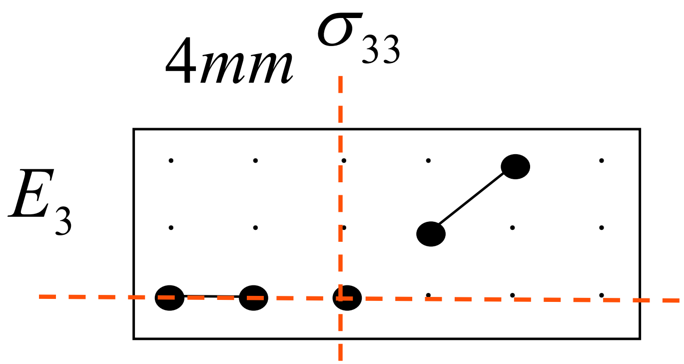
Elastic modules of piezoelectrics

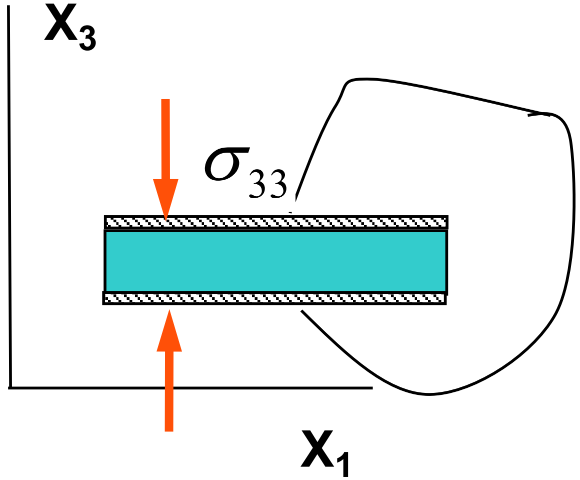
$$D_i = \varepsilon_0 K_{ij} E_j + d_{in}\sigma_n$$

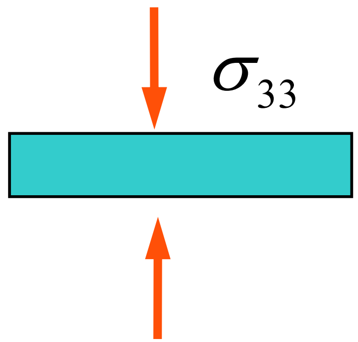
$$\varepsilon_n = d_{in} E_i + s_{nm}\sigma_m$$

$4mm$ $PbTiO_3$

$$\sigma_{33} \neq 0$$







$$D_3 = \varepsilon_0 K_{33} E_3 + d_{333}\sigma_{33}$$

$$D_1 = D_2 = E_1 = E_2 = 0$$

$$E_3 = 0$$

Short-circuit s_{33}

$$D_3 = 0$$

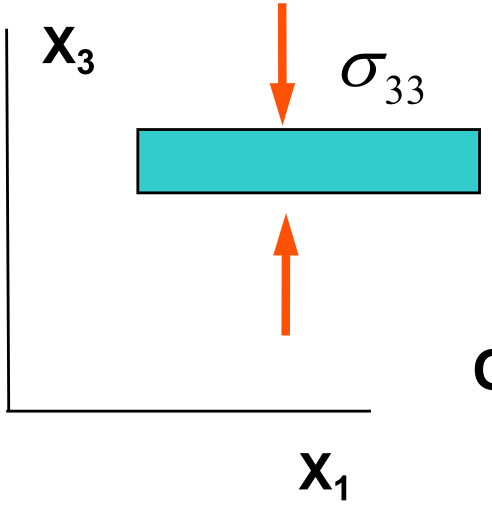
Open-circuit s^*_{33}

Elastic modules of piezoelectrics

$4mm$

$$D_i = \varepsilon_0 K_{ij} E_j + d_{in}\sigma_n$$

$$\varepsilon_n = d_{in}E_i + s_{nm}\sigma_m$$



$$D_3 = 0$$

Open-circuit s^*_{33}

$$D_3 = \varepsilon_0 K_{33} E_3 + d_{333}\sigma_{33}$$

$$0 = \varepsilon_0 K_{33} E_3 + d_{333}\sigma_{33}$$

$$E_3 = -d_{333}\sigma_{33} / \varepsilon_0 K_{33}$$

$$\varepsilon_{33} = -d_{333}^2\sigma_{33} / \varepsilon_0 K_{33} + s_{33}\sigma_{33}$$

$$s^*_{33} = s_{33} - \frac{d_{333}^2}{\varepsilon_0 K_{33}}$$

Effect can be strong (see exercises)

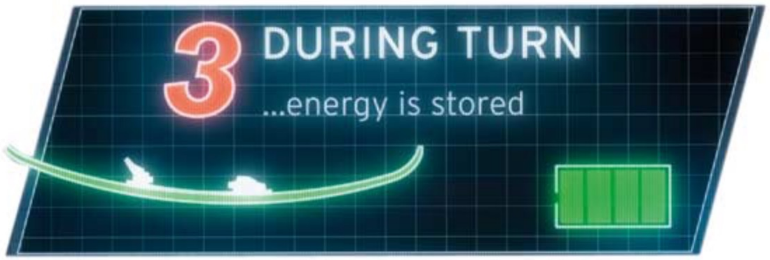
Head: ski intelligence

Ski Dampening System

https://www.head.com/en/sports/ski/technology/emc

Check out the old video of HEAD explaining technology! (see moodle)

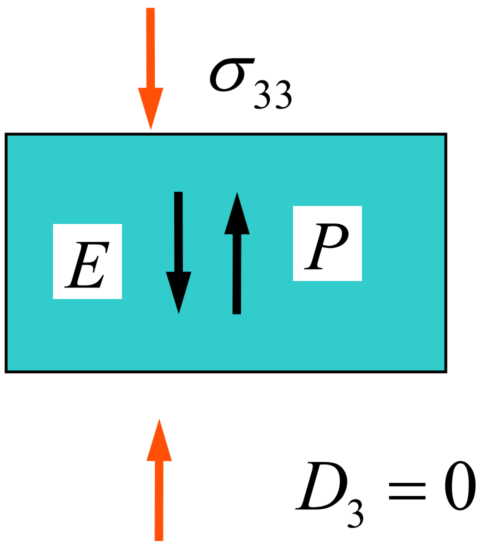
Head says:

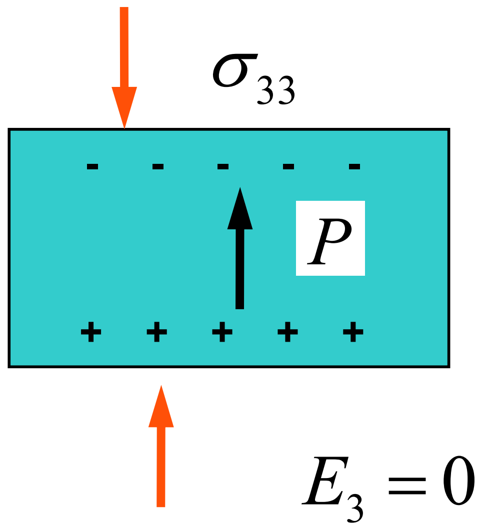


PIEZOELECTRIC FIBERS TRANSFORM KINETIC ENERGY INTO ELECTRICAL ENERGY WHICH IS STORED. ELECTRICAL ENERGY IS IMMEDIATELY RELEASED TO AREAS OF THE SKI, WHERE ADDITIONAL ENERGY IS REQUESTED. TIMING AND RELEASE ARE AUTOMATICALLY CONTROLLED AND COORDINATED. DEPENDING ON THE FLEX PATTERN OF DIFFERENT SKI MODELS, SENSORS ARE PROGRAMMED BEFOREHAND: THE MORE AGGRESSIVE THE SKI HAS TO BE, THE STIFFER THE TAIL WILL BECOME.

Does this make sense? What is the materials physics behind this effect?

Elastic modules of piezoelectrics. Fast = harder; slow = softer





$$s^*_{33} = s_{33} - \frac{d^2_{333}}{\varepsilon_0 K_{33}}$$

harder

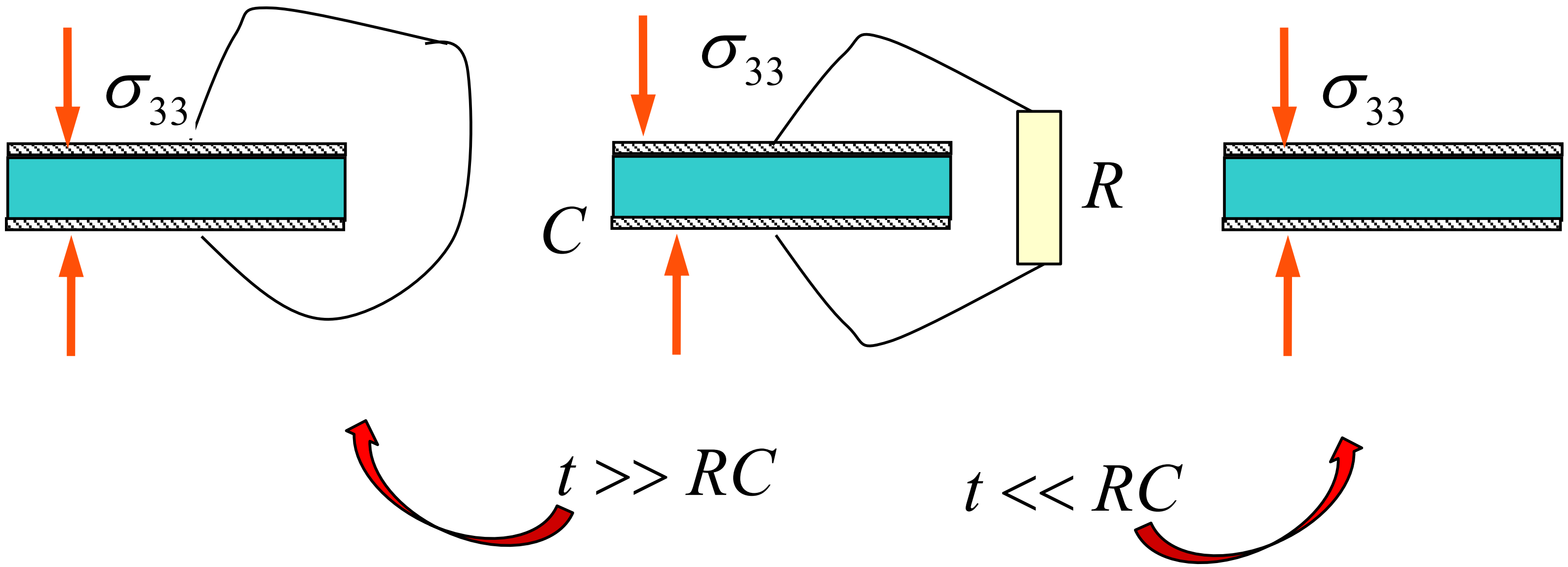
$$s_{33} > s^*_{33}$$

softer

Linear electro-mechanical response under different conditions

Young modulus

Short-circuit ← mixed → Open-circuit



Real-life conditions typically correspond to the "mixed" case, proper choise of *R* determines the time of transition from hard to soft material

Essential

1. Analysis of the energy variations associated with a response enables us to establish new relations (Maxwell relations) between components of the response.

2. Maxwell relations can be either those between components of one tensor or those between tensors controlling different effects.

3. In piezoelectrics, the elastic response is sensitive to the electrical conditions and visa versa.

4. Mechanical boundary condition (clamped/free) and electrical boundary conditions (open/short-circuit) are essential for analyzing the electromechanical response